# High through Put and Enhanced Quaternary Addition Using VLSI 

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#### Abstract

The main objective of this project is to design an enhanced Quaternary Signed Digit (QSD) adder with less propagation delay. Binary number system is easy to implement any arithmetic operations but they have their limitations in the area of circuit complexity and chip area which ultimately increases propagation delay of the circuit. To overcome these limitations QUATERNARY NUMBER SYSTEM is used. This number system decreases chip area about $25 \%$. This project concentrates on the Quaternary Addition. Range of this numbers varies from -3 to 3 . Quaternary addition involves carry free addition which reduces propagation delay.


KEYWORDS: Q.S.A, Latency, Binary addition,

## 1. INTRODUCTION

The major challenges in VLSI design are reducing the area of chip and increasing speed of the circuit. Reducing area can be achieved by optimization techniques and number of instructions executed per second increases as speed increases. The performance of a digital system depends upon performance of adders. Adders are also act as basic building blocks for all arithmetic circuits for example DSP processors. Binary adders are easy to implement because of logic levels involved in it ' 0 ' and ' 1 ', but they have their own limitations in the area of circuit complexity and chip area which ultimately increases propagation delay of the circuit. The drawback of binary adders can be reduced by increasing the range of the numbers used. Signed number system can be used for this purpose. Signed digit numbers allows redundancy of numbers which allow possibility of carry-free addition, but the signed digit number system allow limited carry propagation with some complex addition process which requires large for its implementation. To overcome all these limitations Multi Valued Logic number system is used. It has advantages in many areas as high density along with increasing the speed of operation. One such number system is QSD (Quaternary Signed Digit) number system.
QSD NUMBERS: Quaternary number system is four valued system, it is a base four number system. The QSD number set is $\{\overline{3}, \overline{2}, \overline{1}, 0,1,2$, 3 \} Where $\overline{3}$ represents $-3, \overline{2}$ represents -2 and $\overline{1}$ represents -1. Generalized QSD number D can be represented as
$\mathrm{D}=\sum_{i=0}^{n-1} x^{i} 4^{i}$
Where x can be any value in the range of $\{\overline{3}, \overline{2}$, $\overline{1}, 0,1,2,3\}$. QSD negative number is the complement of QSD positive number.

Example: Advantages of QSD numbers are

1. Circuit complexity reduces in terms of transistor count and interconnections.
2. Occupies less space to store numbers of about $25 \%$ less when compared to Binary Signed Number system.
3. It uses parallelism and reduced gate complexity.
4. Computation speed increases.

ALGORITHM FOR QSD ADDER:
In QSD addition carry propagation chain is eliminated and hence propagation delay decreases which ultimately increases the system speed. The QSD addition range varies from 6 to +6 since the QSD number ranges from -3 to +3 . Below table represents addition of two QSD digits.
TABLE1: Single digit addition result of QSD numbers.

Generalized QSD addition process is done in two steps.
STEP 1: Here intermediate sum and intermediate carry are generated from QSD input digits i.e. addend and augend .
STEP 2: In this step intermediate carry is left shifted by one bit and then combined with intermediate with sum. Decimal numbers from -3 to +3 are represented by one digit QSD number. If decimal number exceeds this range more than one bit of QSD number is required for its representation. For addition of two digits output range varies from -6 to +6 which requires two QSD digits for its representation.MSB represents sum bit and LSB bit represents carry bit. QSD numbers allow redundancy in their number representation i.e. same decimal number can be represented in more than one QSD number. One of such QSD number should be selected as QSD coded number
which prevents further carry rippling during addition. Rules that should be followed to choose a QSD coded number are
RULE 1: Intermediate Sum (IS) should be in the range of -2 to +2 . ( -2 and +2 are included)
IS $=\{-2,-1,0,1,2\}$
RULE 2: Intermediate Carry (IC) should in the range of -1 to +1 . ( -1 and +1 are included)
$\mathrm{IC}=\{-1,0,1\}$
Below table 2 represents the QSD code of a number.
TABLE 2: QSD coded numbers from -6 to +6

| Sum in <br> decimal | QSD <br> representation | QSD coded <br> number |
| :--- | :--- | :--- |
| -6 | $\overline{2} 2, \overline{1} \overline{2}$ | $\overline{1} \overline{2}$ |
| -5 | $\overline{2} 3, \overline{1} \overline{1}$ | $\overline{1} \overline{1}$ |


|  | -3 | -2 | -1 |  | 0 | 2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3 | -6 | -5 | -4 | -3 | -2 | -1 | 0 |
| -2 | -5 | -4 | -3 | -2 | -1 | 0 | 1 |
| -1 | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
| 0 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| 1 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| 2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
| 3 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|  | -4 |  | $\overline{1} 0$ |  |  | $\overline{1} 0$ |  |
|  | -3 |  | $\overline{1} 1,0 \overline{3}$ |  |  | $\overline{1} 1$ |  |
|  | -2 |  | $\overline{1} 2,0 \overline{2}$ |  |  | $0 \overline{2}$ |  |
|  | -1 |  | $\overline{1} 3,0 \overline{1}$ |  |  | $0 \overline{1}$ |  |
|  | 0 |  | 00 |  |  | 00 |  |
|  | 1 |  | 01, $1 \overline{3}$ |  |  | 01 |  |
|  | 2 |  | 02, $1 \overline{2}$ |  |  | 02 |  |
|  | 3 |  | 03, $1 \overline{1}$ |  |  | $1 \overline{1}$ |  |
|  | 4 |  | 10 |  |  | 10 |  |
|  | 5 |  | 11, $2 \overline{2}$ |  |  | 11 |  |
|  | 6 |  | 12, $2 \overline{3}$ |  |  | 12 |  |

The first column of the table represents decimal number, second column of the table shows the available multiple QSD representations of given decimal number and third column represents QSD coded number from above mentioned rules. Hence the addition of two QSD numbers is performed in two steps and further carry rippling is eliminated by using two rules for IC and IS. Addition process can be understood easily by using following examples

Ex1: ADD two numbers 40 and 40 in quaternary number system.
$40_{10}=220_{4}$

| $\pm 2$ | 2 | 0 |
| :---: | :---: | :---: |
| 2 | 2 | 0 |
| 4 | 4 | 0 |
| 10 | 10 | $\frac{1}{80}$ |
| IC: 110 |  |  |

$\frac{\text { STEP 2: }}{11} \quad 0 \longrightarrow$ IC shifted left bye one bit
$\begin{array}{llll}0 & 0 & 0 & \longrightarrow\end{array}$


ADD two numbers (negative example): 130 and 200.
$130_{10}=2002_{4}$
$-200_{10}=\overline{3} 0 \overline{2} 0_{4}$
STEP 1:


IC : 0000 IS : $\overline{1} 0 \overline{2} 2$
$\frac{\text { STEP 2: }}{0 \quad 0} 0$

$$
15: 1022
$$

$\begin{array}{llll}0 & 0 & 0 & 0\end{array} \underset{\text { one bit }}{\longrightarrow}$ IC shifted left by

$$
\overline{1} \quad 0 \quad \overline{2} \quad 2 \quad \text { IS }
$$



## 2. IMPLEMENTATION PROCEDURE

 ADDITION PROCEDURE FOR SINGLE BIT:Addition of two single digit QSD numbers which ranges from -3 to +3 produces output in the range of -6 to +6 . As this is out of range of input two bits are required to represent it. Lower significant bit serves as sum and most significant bit serves as carry bit. This addition process is done by converting the input into signed 2 'scomplementrepresentation and all output combinations are

| INPUT |  |  | OUTPUT |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| QSD |  | BINARY | $\begin{aligned} & \text { DECIMA } \\ & \text { L } \\ & \hline \text { sum } \end{aligned}$ | QSD |  | BINARY |  |
| Ai | Bi | Ai Bi |  | Ci | Si | Ci | Si |
| 3 | 3 | 011011 | 6 | 1 | 2 | 001 | 010 |
| 3 | 2 | 011010 | 5 | 1 | 1 | 001 | 001 |
| 2 | 3 | 010011 | 5 | 1 | 1 | 001 | 001 |
| 3 | 1 | 011001 | 4 | 1 | 0 | 001 | 000 |
| 1 | 3 | 001011 | 4 | 1 | 0 | 001 | 000 |
| 2 | 2 | 010010 | 4 | 1 | 0 | 001 | 000 |
| 1 | 2 | 001010 | 3 | 1 | -1 | 001 | 111 |
| 2 | 1 | 010001 | 3 | 1 | -1 | 001 | 111 |
| 3 | 0 | 011000 | 3 | 1 | -1 | 001 | 111 |
| 0 | 3 | 000011 | 3 | 1 | -1 | 001 | 111 |
| 1 | 1 | 001001 | 2 | 0 | 2 | 000 | 010 |
| 0 | 2 | 000010 | 2 | 0 | 2 | 000 | 010 |
| 2 | 0 | 010000 | 2 | 0 | 2 | 000 | 010 |
| 3 | -1 | 011111 | 2 | 0 | 2 | 000 | 010 |
| -1 | 3 | 111011 | 2 | 0 | 2 | 000 | 010 |
| 0 | 1 | 000001 | 1 | 0 | 1 | 000 | 001 |
| 1 | 0 | 001000 | 1 | 0 | 1 | 000 | 001 |
| 2 | -1 | 010111 | 1 | 0 | 1 | 000 | 001 |
| -1 | 2 | 111010 | 1 | 0 | 1 | 000 | 001 |
| 3 | -2 | 011110 | 1 | 0 | 1 | 000 | 001 |
| -2 | 3 | 110011 | 1 | 0 | 1 | 000 | 001 |
| 0 | 0 | 000000 | 0 | 0 | 0 | 000 | 000 |
| 1 | -1 | 001111 | 0 | 0 | 0 | 000 | 000 |
| -1 | 1 | 111001 | 0 | 0 | 0 | 000 | 000 |
| 2 | -2 | 010110 | 0 | 0 | 0 | 000 | 000 |
| -2 | 2 | 110010 | 0 | 0 | 0 | 000 | 000 |
| -3 | 3 | 101011 | 0 | 0 | 0 | 000 | 000 |
| 3 | -3 | 011101 | 0 | 0 | 0 | 000 | 000 |
| 0 | -1 | 000111 | -1 | 0 | -1 | 000 | 111 |
| -1 | 0 | 111000 | -1 | 0 | -1 | 000 | 111 |
| -2 | 1 | 110001 | -1 | 0 | -1 | 000 | 111 |
| 1 | -2 | 001110 | -1 | 0 | -1 | 000 | 111 |
| -3 | 2 | 101010 | -1 | 0 | -1 | 000 | 111 |
| 2 | -3 | 010101 | -1 | 0 | -1 | 000 | 111 |
| -1 | -1 | 111111 | -2 | 0 | -2 | 000 | 110 |
| 0 | -2 | 000110 | -2 | 0 | -2 | 000 | 110 |
| -2 | 0 | 110000 | -2 | 0 | -2 | 000 | 110 |
| -3 | 1 | 101001 | -2 | 0 | -2 | 000 | 110 |
| 1 | -3 | 001101 | -2 | 0 | -2 | 000 | 110 |
| -1 | -2 | 111110 | -3 | -1 | 1 | 111 | 001 |
| -2 | -1 | 110111 | -3 | -1 | 1 | 111 | 001 |
| -3 | 0 | 101000 | -3 | -1 | 1 | 111 | 001 |
| 0 | -3 | 000101 | -3 | -1 | 1 | 111 | 001 |

tabulated in below table3.
TABLE 3: Addition result conversion from QSD to signed binary 2's complement form for single digit QSD addition.

The minterms for the intermediate carry $\left(\mathbf{I C}_{2}\right.$, $\mathrm{IC}_{1}, \mathrm{IC}_{0}$ ) are:
IC ${ }_{2}=a_{2} b_{2}\left(\mathbf{a}_{0} b_{0} a_{1} b_{1)+} \overline{\left(a_{1}+b_{1}\right)}\right)\left(\mathbf{a}_{2} \bar{b}_{0}+b_{2} \bar{a}_{0}\right)$
IC ${ }_{1}=a_{2} b_{2} \underline{\left(a_{0} b_{0} a_{1} b_{1)+}+\left(a_{1}+b_{1}\right)\left(a_{2} b_{0}+b_{2} a_{0}\right)\right.}$

IC ${ }_{0}=$ IC $_{2}+\mathbf{a}_{2} b_{2}\left(\overline{a_{1} b_{1}+b_{1}} b_{0}+b_{0} a_{1}+b_{1} a_{0}+a_{1} a_{0}\right)$ Minterms for intermediate sums are:
$\mathrm{IS}_{0}=\mathbf{a}_{0} \bar{b}_{0}+\overline{\mathbf{a}}_{0} \mathbf{b}_{0}$
IS $\mathbf{I}_{1}=\left(a_{1} \bar{b}_{1}+\bar{a}_{1} \mathbf{b}_{1}\right) a_{0} \bar{b}_{0} \pm\left(a_{1}, \bar{b}_{1}+\bar{a}_{1} b_{1}\right) a_{0} b_{0}$
IS $_{2}=$ IS ${ }_{0}\left(\overline{\mathbf{a}}_{1} \mathbf{b}_{1}+\mathrm{a}_{1} \bar{b}_{1}\right)+\mathrm{b}_{2} \overline{\mathbf{a}}_{1} \bar{b}_{0}+\overline{\mathbf{a}}_{2} \bar{b}_{1} \mathrm{a}_{0}+\mathrm{a}_{0} \mathrm{~b}_{0}$

$$
\bar{a}_{1} \bar{b}_{1}\left(\mathbf{a}_{2}+\mathbf{b}_{2}\right)
$$

The final sum which is carry free is generated from those outputs i.e. Intermediate carry $\left(\mathrm{IC}_{2}, \mathrm{IC}_{1}\right.$, and $\mathrm{IC}_{0}$ ) and Intermediate sum ( $\mathrm{IS}_{2}, \mathrm{IS}_{1}$, and $\mathrm{IS}_{0}$ ). Therefore it has six input and three output bits.
$\mathbf{S}_{\mathbf{0}}=\mathbf{I C}_{0} \overline{\mathbf{S}}_{\mathbf{0}}+\mathrm{IC}_{\mathbf{0}} \mathrm{IS}_{0}$
$\mathrm{S}_{1}=\mathrm{IC}_{1} \oplus \mathrm{IS}_{1} \oplus \mathrm{IC}_{0} \mathrm{IS}_{0}$

Input augend $A$ is represented by 3 variables $A_{0}$, $\mathrm{A}_{1}, \mathrm{~A}_{2}$ and addend B is represented by 3 variables $\mathrm{B}_{0} \quad, \mathrm{~B}_{1}, \quad \mathrm{~B}_{2}$. Outputs intermediate carry(IC), intermediate sum(IS) are represented by 3 variables $\mathrm{IC}_{0}, \mathrm{IC}_{1}, \mathrm{IC}_{2}$ and $\mathrm{IS}_{0}, \mathrm{IS}_{1}, \mathrm{IS}_{2}$ simultaneously. As IC can be represented using only 2 bits third bit is allotted same value as second bit. The six variable expressions for IC, IS and final sum can be resolved from above table using k-map reduction. Verilog code for IC, IS and SUM has been written and simulated using XILINX 14.7 simulator and then synthesized using SYNPOSIS DESIGN COMPILER.

## 3. ADDITION PROCEDURE FOR

 MORE THAN 2 DIGITS:N bit QSD addition is done in 2 steps. First step IC and IS are generated and in second step IC is left shifted by one digit and then added with IS using sum equations solved from above table. Further carry rippling is eliminated by two rules as mentioned above in section 2 . These limitations over IC and IS restricts sum from overflow and therefore final result will be carry free. respectively. As illustrated in this proposed decomposition method achieves higher printability because of the following

## 4. RESULTS

In this paper low power QSD ADDER unit is simulated using Modelsim and synthesizeed using XILINX Design Compiler in
Operating Condition Name: TYPICAL
Wire Loading Model Mode: top.

| -3 | -1 | 101 | 111 | -4 | -1 | 0 | 111 | 000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| -1 | -3 | 111 | 101 | -4 | -1 | 0 | 111 | 000 |
| 2 | -2 | 110 | 110 | -4 | -1 | 0 | 111 | 000 |
| -2 | -3 | 101 | 110 | -5 | -1 | -1 | 111 | 111 |
| -3 | -2 | 110 | 101 | -5 | -1 | -1 | 111 | 111 |
| -3 | -3 | 101 | 101 | -6 | -1 | -2 | 111 | 110 |

Simulation and layout designs are listed out as shown in below mentioned


OUTPUT OF SIMULATION


## 5. CONCLUSION

Finally, this project concludes that Quaternary signed digit adder for single digit addition is designed with low latency, power optimization and more accuracy. The dynamic power dissipation for designed project is 35.6781 at 13 GHz frequency. The delay of single digit adder is 0.73 ns . This design is simulated using MODELSIM and synthesized using Xilinx Design Compiler.

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